Mathematical analysis I

Course title	**Mathematical analysis I**
Course code	Mate1009
Branch of science	Mathematics
Sub-branch	Differential Calculus
Credit points	6
ECTS credit points	9
Total contact hours Lectures Seminars	96 48 48

Prerequisites (course title, part of the program)

Course abstract
This course is designed for Bachelor study program "Mathematics" students. The aim of the course is to introduce students to the calculus of one real variable and their applications. Classification of the functions, continuity and differentiability of a function of one real variable, fundamental theorems of calculus and their applications will be considered.

Results
• be able to calculate limits; • be able to differentiate the functions of one real variable; • apply derivatives to solve geometrical and physical problems.

Course content:
Lectures – 48 contact hours, seminars – 48 contact hours. The set of real numbers R and its geometric interpretation. Bounded and unbounded sets. Intervals and neighbourhoods. A real function of one real variable. Domain and range of a function, function’s graph. Operations with functions. Function composition. Classification of the functions (monotone, even and odd, periodic, bounded functions). Sequences and subsequences. Invertible function. Inverse function. Limit of a function. Limit of a sequence. Trigonometric limits. Limit properties. One-sided limits. Bolzano–Weierstrass theorem. Cauchy criterion. Continuity of a function at a point. Continuity of sum, product and quotient of functions. Continuity of function composition. One-sided continuity. Discontinuity points and their classification. Intermediate Value Theorem. Extreme value theorem. Derivative of a function, differential. Geometric and mechanical interpretation of the function derivative. Differentiation formulas: sum, product and quotient rules. Basic formulas of derivatives. Higher order derivatives and differentials. Mechanical interpretation of the second-order derivative of a function. Differentiation of a function defined parametrically. Fermat’s Theorem. Rolle’s Theorem. Lagrange Mean Value Theorem. Cauchy’s Mean Value Theorem. L’hopital’s rule. Taylor’s formula. Constant, increasing or decreasing functions. Functions extreme points, local maximum and local minimum of a function. Convex and concave functions. Inflection points. Complete study of a function, plotting graph of a function.

Course plan:

Lecture topics

1. The set of real numbers R. Geometric interpretation of real numbers. Bounded and unbounded sets. Intervals and neighbourhoods.
2. A real function of one real variable. Domain and range of a function, function's graph. Operations with functions. Function composition.
3. Classification of the functions (monotone, even and odd, periodic, bounded functions). Sequences and subsequences. Arithmetic and geometric progression.
4. Invertible function. Inverse function. Theorem on an inverse function of a monotone function.
5. Limit of a function. Limit of a sequence. (General definition and several cases).
6. Trigonometric limits. Theorem on uniqueness of limits.
7. Limit properties: limit of a sum, product and quotient of functions.
8. Limit of function composition. Inequalities and limits.
9. One-sided limits. Infinitely-small and infinitely-large functions, their comparison.
10. Limit of a monotone sequence.
11. The limit definition of e (Euler's number).
12. Bolzano–Weierstrass theorem. Cauchy criterion.
13. Continuity of a function at a point. Continuity of sum, product and quotient of functions.
14. Continuity of function composition. One-sided continuity. Discontinuity points and their classification. Asymptotes of the function graph.
15. Limit of monotonous function. Intermediate Value Theorem.
16. Continuity of inverse function. Boundedness of continuous function on a closed and bounded interval. Extreme value theorem. Uniform continuity.
17. Derivative of a function, differential. Geometric and mechanical interpretation of the function derivative. Continuity of the differentiable function.
18. Differentiation formulas: sum, product and quotient rules. Basic formulas of derivatives.
19. Higher order derivatives and differentials. Mechanical interpretation of the second-order derivative of a function. Differentiation of a function defined parametrically.
20. Fermat's Theorem. Rolle's Theorem. Lagrange Mean Value Theorem. Cauchy's Mean Value Theorem.
21. L'hopital's rule. Taylor's formula.
22. Constant, increasing or decreasing functions. Functions extreme points, local maximum and local minimum of a function.
23. Necessary and sufficient conditions of function extrema. Finding functional global extrema.
24. Convex and concave functions. Inflection points.

Seminar topics

1. The absolute value of a real number. Absolute value equations and inequalities. Graph of modulus function.
2. Domain and range of real function of one real variable.
3. Classification of the functions of one real variable.
4. Equal functions. Inverse function.
5. Definition of limit of a function and sequence.
6. Undetermined forms of limits „$\frac{0}{0}$”, „$\frac{\infty}{\infty}$”.
7. Undetermined forms of limits “$0 \cdot \infty$”, “$\infty - \infty$”.
8. Trigonometric limits.
9. Limits calculation.
10. One-sided limits. Infinitely-small and infinitely-large functions.
11. Limits with number e.
12. Undetermined forms of limits “1^{∞}”, “∞^{0}”, “0^{0}”.
13. Limits calculation.
14. Continuity of a function. Discontinuity points and their classification.
15. Asymptotes of the function graph.
16. Differentiation of a sum, product and quotient of the functions .
17. Differentiation of function composition.
18. Function differentiation.
19. Higher order derivatives and differentials.

20. Limits calculation using the L'hopital's rule.
21. Investigating monotonicity of a function.
22. Finding local and global extreme of functions.
23. Investigating concavity of a function.
24. Complete study of a function, function graph ploting.

Independent work of students:
The course provided a systematic control tasks solving. During semester students must complete 6 independent works. For each student tasks are individual. All entries must be satisfied and positively evaluated by the beginning of the session.

Course requirements:

Acquisition and presentation of knowledge and skills described within the course.
Final evaluation form for the course – pass, exam.
Course requirements – regular attendance and active work in 60%, independent work execution of 40%.
Study methods and forms – lectures, seminars, independent work.

Mācību pamatiteratūra :

1. M.L. Bittinger, D.J. Ellenbogen, S. Surgent. Calculus and Its Applications, Addison Wesley, 2011.
2. D. Brannan. A First Course in Mathematical Analysis, Cambridge University Press, 2006.
3. I. Bula, I. Buls. Matemātiskā analīze ar ģeometrijas un algebras elementiem. I daļa, Zvaigzne ABC, 2003.
4. J.V. Deshpande. Mathematical Analysis and Applications : An introduction, Alpha Science International Ltd, 2004.
5. V. Gedroics. Ievads matemātiskajā analīzē (2003.) http://de.du.lv/matematika/ievmatanavit.pdf
6. V. Gedroics. Viena argumenta funkciju diferenciālrēķini (2002.) http://de.du.lv/matematika/fun1.pdf
7. V. Gedroica. Ievads matemātiskajā analīzē (2003) http://de.du.lv/matematika/vallievads_col.pdf
8. V. Gedroica. Viena argumenta funkciju diferenciālrēķini (2005) http://de.du.lv/matematika/gedroica/Difrek1.pdf
9. M. Giaquinta, G. Modica. Mathematical Analysis : Functions of One Variable, Birkhauser, 2003.
10. P.D. Lax, M.S. Terrell. Calculus With Applications, Springer, 2013.
11. M.A. Robdera. A Concise Approach to Mathematical Analysis, Springer, 2003.
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Papildliteratūra:

1. Dz. Bože, L. Biezā, B. Siliņa, A. Strence. Uzdevumu krājums augstākajā matemātikā, Zvaigzne, 1986.
2. A. Browder. Mathematical Analysis : An Introduction, Springer, 2001.
3. A. Cibulis. Ekstrēmu uzdevumi. 1. daļa, Rīga, 2003.
4. V. Gedroica, V. Gedroics. Elementārās funkcijas, Daugavpils, 1988.
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6. M. Friedman, A. Kandel. Calculus Light, Springer, 2011.
7. C.C. Pugh. Real Mathematical Analysis, Springer, 2002.
8. E. Kronbergs, P. Rivža, Dz. Bože. Augstākā matemātika. 1. daļa, Zvaigzne, 1988.
9. E. Mendelson. Schaum's 3,000 solved problems in calculus, MGH, 2009.
10. V.A. Zorich. Mathematical Analysis 1, Springer, 2004.
11. Задачник по курсу математического анализа. Под ред. Н.Я.Виленкина. Ч. 1-2, Просвещение, 1971.
12. Ильин В.А., Садовничий В.А., Сендов Бл.Х. Математический анализ, Наука, 1979.

13. Кудрявцев Л.Д. Курс математического анализа. Ч. I, Высшая школа, 1988.
14. Лунгу К.Н., Письменный Д.Т., Федин С.Н., Шевченко Ю.А.. Сборник задач по высшей математике. 1 курс, Москва, Айрис-пресс, 2008.
15. Райков Д.А. Одномерный математический анализ, Высшая школа, 1982.
16. Уваренков И.М., Маллер М.З. Курс математического анализа. Т. 1., Просвещение, 1966; т. 2, Просвещение, 1976

Periodika un citi informācijas avoti:

1. WolframAlpha (Calculus & Analysis) https://www.wolframalpha.com/examples/Calculus.html
2. Высшая математика http://www.math24.ru/
3. Математический анализ http://www.exponenta.ru/educat/class/courses/ma/theme1/theme.asp
4. В. Дятлов. Математический анализ http://matan.dyatlov.org/

Piezīmes: