Course title	**Linear algebra I**
Course code	Mate1011
Branch of science	Mathematics
Sub-branch of science	Algebra and Mathematical Logic
Credit points	3
ECTS credit points	4.5
Total contact hours Number of hours for lectures Number of hours for seminars and practical work	48 16 32

Course developer (-s)
Ph.D., leading researcher Peteris Daugulis.

Prerequisite knowledge (course name)
None

Course abstract:
The course is designed for the students of the bachelor study program „Mathematics”. The goal of the course is to cover the introductory chapters of linear algebra – number fields, basic matrix algebra, systems of linear equations, linear spaces and applications.

Learning outcomes:
• Knowledge of matrix algebra; ability to solve the appropriate level tasks - standard textbook exercises on matrix properties and operations. • Knowledge of methods for eliminating solving linear equation systems (Gauss method, etc.); ability to solve linear equation systems. • Knowledge of the basic concepts of linear space; ability to solve the corresponding tasks - standard textbook exercises on linear independence, base and dimension.

Course content:
Lectures – 16 CH, seminars – 32 CH 1. Sets, number rings and fields. Important notions: set, Euler diagram, set equality, subset, power set, complement, union, intersection, set difference, Cartesian product, number rings and fields, field of complex numbers, arithmetic operations of complex numbers. Important facts and methods: properties of subsets, properties of set operations, methods for proving set equality. 2. Matrix algebra. Important notions: matrix, row, column, main diagonal, row/column matrix, square matrix, zero matrix, basis matrix, identity matrix, diagonal matrix, triangular matrix, symmetric/antisymmetric matrix, block matrix, transposing, matrix addition, matrix

multiplication by a number, linear combination of matrices, matrix multiplication.

Important facts and methods: properties of matrix operations.

3. Linear systems of equations (LSE).

Important notions: linear equation, LSE, LSE pictures - system, matrix and column picture (notation), LSE solution set, elementary operations od LSE, matrices of elementary operations, row echelon form of a matrix, normalized row echelon form, main variables, free variables, general solution, particular solution.

Important facts and methods: properties of elementary operations, realization of elementary operations via matrix multiplication, algorithm for transformation of a matrix into a row echelon form, Gauss elimination method, algorithm for transformation of a matrix into the Hermite form, Gauss-Hermite elimination method, properties of solutions of LSE.

4. Elementary column operations (ECO), normal form, ranks

Important notions: ECO and their matrices, the normal form of a matrix, row rank, column rank, rank.

Important facts and methods: properties of ECO, algorithm for transformation of a matrix into the normal form, uniqueness of the Hermite form, Kronecker-Capelli theorem, EO preserve both ranks, equality of ranks, uniqueness of the normal form, properties of matrix rank.

5. Matrix inversion.

Important notions: left/right inverse matrix, inverse matrix of a square matrix.

Important facts and methods: inverse matrix existence criterion, properties of invertible matrices, fundamental theorem of invertible matrices, square LSE with invertible system matrices, algorithm for inversion of square matrices.

6. Determinant and its applications.

Important notions: determinant, recursive definition of determinant (expansion along the first column), adjugate matrix.

Important facts and methods: determinants of elementary matrices, properties of determinant, determinant formulas for small matrices, triangulation method, expansion along a row or column, Cramer formula method, geometric interpretation of determinant.

7. Linear spaces (LS).

Important notions: k-linear space, arithmetic (vector) LS, matrix LS, function LS, null-space of a matrix, linear subspace, sum of subspaces, linear combination, linear closure (span), generating set, liner dependence/independence, basis of a LS, finitely generated LS, dimension.

Important facts and methods: basic properties of LS, examples of LS, properties of subspaces, properties of linear span, properties of linear dependence, canonical bases, existence of bases for finitely generated LS, properties of basis.

8. Properties and applications of bases and dimension.

Important notions: internal direct sum of subspaces, complementary subspace, row and column space of a matrix.

Important facts and methods: algorithms for computing element coordinates after a basis change, theorem about dimensions of subspaces, properties of complementary subspaces, interpretation of matrix ranks, criterions of matrix invertibility.

9. Fundamentals of relations, quotient space.

Important notions: binary relation, special cases of relations (reflexive, symmetric, antisymmetric, transitive), partial order, equivalence, subspace coset, coset operations, quotient space.

Important facts and methods: properties of cosets, interpretation of LSE solution sets in terms of cosets, well definiteness and properties of coset operations, method for finding a quotient space basis, quotient space dimension formula.

Course plan:

Lectures - 16 CH, seminars - 32 CH.

Lecture topics:

1. Set theory, number rings and fields.
2. Introduction to matrix algebra.
3. Linear systems of equations (LSE), elementary operations, LSE solving methods – Gauss and Gauss-Hermite elimination methods.
4. Matrix rank and normal form, matrix inversion.
5. Introduction to determinant theory.
6. Properties and applications of determinants. Permutations.
7. Linear spaces, linear dependence, independence, basis, dimension.
8. Applications of linear spaces in matrix algebra and LSE solving.

Seminar topics:

1. Set theory.
2. Number rings and fields.
3. Matrix algebra.
4. Elementary operations of LSE and their realization via matrices.
5. LSE solving using Gauss and Gauss-Hermite elimination.
6. Matrix rank and normal form.
7. Matrix inversion.
8. Computation of determinant.
9. Properties and applications of determinant.
10. Linear spaces (LS), linear dependence, independence.
11. Basis and dimension of LS.
12. Basis chance, properties of subspaces.
13. Relations, quotient space and its applications.
14. Applications of LSE.
15. Applications of LSE, matrix-graph duality.
16. Review.

Independent work of students:

Each student must solve at least 10 homeworks, which correspond to lecture topics. Every homework consists of 4-6 mandatory problems and 2-3 optional (hard) problems. Students are offered topics for presentations, students can prepare presentations and earn extra points for final evaluation.

Requirements for awarding credit points:

Acquiring and presenting skills, abilities and knowledge, gained within the course.
Types of assessment - homework, test, exam.
Other requirements – homework - 40%, test - 20%, exam - 30%, increased difficulty task solving - 10%.
Study methods and forms used – seminars, consultations, independent work, discussion, argumentation.

Compulsory reading :

1. P.Daugulis. Lekcijas lineārajā algebrā, DU, www.moodle.du.lv, 2011.
2. T.S.Blyth, E.F.Robertson. Basic linear algebra, Springer, 2002.
3. D.J.S.Robinson. A course in linear algebra with applications, WS, 2006.

Further reading:

1. A.Galiņš. Lineāru vienādojumu sistēmas un vektoru telpas (lekciju konspekts), DU.
2. A. Ozerskis, Z. Ozerska. Uzdevumi algebrā un skaitļu teorijā, Daugavpils, DPI, 1983.
3. L.J.Kulikov. Algebra i teorija čisel, Visšaja škola, 1979. (krievu val.)
4. J.S.Ljapin, A.J.Jevsejev. Algebra i teorija čisel, Nauka, 1978. (krievu val.)
5. D.K.Faddejev, I.S.Sominskij. Sbornik zadač po visšei algebre, Nauka, 1977. (krievu val.)
6. A.I.Kostrikin, J.I.Manin. Lineinaja algebra i geometrija, Nauka, 1986. (krievu val.)
7. S.Axler. Linear algebra done right, Springer, 2004. (angļu val.)
8. D.Poole. Linear algebra – a modern introduction, Thomson Brooks/Cole, 2006. (angļu val.)

Periodicals and other sources:

1. www.wikipedia.org.

Notes:

Students are provided with lecture materials in electronic form, which they can use during lectures. The workshops prioritize homework assignments, the solutions of which students must complete at home.