Course title	**Linear algebra II**
Course code	Mate1014
Branch of science	Mathematics
Sub-branch of science	Algebra and Mathematical Logic
Credit points	3
ECTS credit points	4.5
Total contact hours Number of hours for lectures Number of hours for seminars and practical work	48 16 32

Course developer (-s)
Ph.D., leading researcher Peteris Daugulis.

Prerequisite knowledge (course name)
Mate1011

Course abstract:
The course isdesigned for the students of the bachelor study program „Mathematics". The goal of the course is to cover the following chapters of linear algebra – linear maps and operators, Eigen-analysis, inner product spaces, applications of linear algebra in numerical mathematics (least squares method and linear optimization).

Learning outcomes:

- Knowledge of the theory of linear representations; ability to solve tasks of a corresponding level - standard textbook exercises on linear representations, their matrices, structure and qualities.
- Knowledge of the theory of Euclidean space; ability to solve standard school textbook exercises on scalar, Graham-Schmitz algorithms, Orto projects.
- Knowledge of the following linear algebra applications: solving the linear equation systems with the least squares method, solving linear optimization problems; ability to deal with appropriate level tasks.

Course content:

Lectures – 16 CH, seminars – 32 CH

1. Linear mappings (LM).

Important notions: k-linear mapping, isomorphism, operator, functional, distinguished LM – zero, identity, subspace inclusion, element defining, projection, examples of LM in vector and matrix spaces, matrix of LM with respect to the given bases, image and kernel of a LM, operations with LM – sum, scalar multiplication, composition, matrix changes for LM, linear isomorphism (LI).

Important facts and methods: main properties of LM, the matrix formalism of LM, kernel and image dimension property, properties of LI, dimension, isomorphism of LS with k^n.

2. Structure of LM and LO, eigenvalues and eigenvectors.

Important notions: invariant subspace of LO, eigenvector, eigenvalue, characteristic polynomial of LO and matrix, similar matrices, diagonalizable matrix.

Important facts and methods: LM structure theorem, properties of invariant subspaces, equivalent definitions of eigenvalues, algorithms for finding eigenvalues and eigenvectors, invariance of characteristic polynomial, properties of eigenvectors, Gershgorin theorem in the real and complex cases, Perron theorem, diagonalizibility criterions.

3. Euclidean spaces.

Important notions: bilinear form, symmetric form, matrix of bilinear form, scalar product, Euclidean space, orthogonal elements, Euclidean norm, orthonormal set/basis, orthogonal complement, complementing subspace, isometry, orthogonal matrix, projection.

Important facts and methods: bilinear form is determined on basis elements, bilinear matrix basis change, properties of Euclidean norm, linear independence of orthogonal set, Gram-Schmigt algorithm, coordinates and norm with respect to orthonormal basis, properties of orthogonal complement, extremal property of orthogonal projection, spectral theorem of symmetric matrices.

4. Approximate solving of LSE using the least square method.

Important notions: residue vector, approximate solution of LSE, least square solution, optimal least square solution, normal system of LSE.

Important facts and methods: solutions of normal systems as least square solution, solvability of normal systems, finding optimal solutions of normal systems.

5. Solving systems of linear inequalities over R.

Important notions: system of linear inequalities (SLI), general and matrix representation of SLI, SLI solution set, elementary transformations of SLI, normal forms of SLI.

Important facts and methods: elimination of one variable, Fourier-Motzkin method, properties of conviena unknown turn off, Fourier-Motzkin method.

6. Linear optimization problem.

Important notions: optimization, goal function, optimal solution, optimization problem, linear domain, linear programming problem (LPP), LPP normal forms, elementary transformations of LPP, fundamental solutions of LPP.

Important facts and methods: LPP conversions into normal forms, optimal solutions, graphic and simplex methods for solving LPP.

Course plan:

Lectures - 16 CH, seminars - 32 CH.

Lecture topics:

1. LM, definitions and basic facts.
2. Properties of LM, basis change.
3. Linear isomorphisms, structure of LM, eigenvalues and eigenvectors, their properties and uses.
4. Bilinear forms, scalar product.
5. Euclidean spaces, properties, mappings, orthogonal projection.
6. Spectral theorem of symmetric matrices, other classic decompositions and normal forms.
7. Approximate solving of systems of linear equations.
8. Systems of linear inequalities, linear optimization, solution methods for linear optimization problems.

Seminar topics:

1. LM, matrix formalism.
2. Image and kernel of LM.
3. Basis change and LM.
4. Normal form of LM, characteristic polynomial, eigenvectors and eigenvalues.
5. Properties and uses of eigenvectors and eigenvalues – Gershgorin and Perron theorems.
6. Diagonalisation.
7. Bilinear forms and their matrices.
8. Gram-Schmidt algorithm.
9. Properties of Euclidean spaces, orthogonal complement.
10. Orthogonal projection.
11. Properties of matrix fundamental subspaces.
12. Spectral theorem of symmetric matrices.
13. Approximate solutions of linear systems.
14. Systems of linear inequalities, linear optimization.
15. Methods for solving linear optimization problems, simplex method.
16. Review.

Independent work of students:

Each student must solve at least 10 homeworks, which correspond to lecture topics. Every homework consists of 4-6 mandatory problems and 2-3 optional (hard) problems. Students are offered topics for presentations, students can prepare presentations and earn extra points for final evaluation.

Requirements for awarding credit points:

Acquiring and presenting skills, abilities and knowledge, gained within the course.
Types of assessment - homework, test, exam.
Other requirements – homework - 40%, test - 20%, exam - 30%, increased difficulty task solving - 10%.
Study methods and forms used – seminars, consultations, independent work, discussion, argumentation.

Compulsory reading :

1. P.Daugulis. Lekcijas lineārajā algebrā, DU, www.moodle.du.lv, 2011.
2. T.S.Blyth, E.F.Robertson. Basic linear algebra, Springer, 2002. (angļu val.)
3. D.J.S.Robinson. A course in linear algebra with applications, WS, 2006. (angļu val.)

Further reading:

1. A.Galiņš. Lineāru vienādojumu sistēmas un vektoru telpas (lekciju konspekts), DU.

2. A. Ozerskis, Z. Ozerska. Uzdevumi algebrā un skaitļu teorijā, Daugavpils, DPI, 1983.
3. L.J.Kulikov. Algebra i teorija čisel, Visšaja škola, 1979. (krievu val.)
4. J.S.Ljapin, A.J.Jevsejev. Algebra i teorija čisel, Nauka, 1978. (krievu val.)
5. D.K.Faddejev, I.S.Sominskij. Sbornik zadač po vissei algebre, Nauka, 1977. (krievu val.)
6. A.I.Kostrikin, J.I.Manin. Lineinaja algebra i geometrija, Nauka, 1986. (krievu val.)
7. S.Axler. Linear algebra done right, Springer, 2004. (angļu val.)
8. D.Poole. Linear algebra – a modern introduction, Thomson Brooks/Cole, 2006. (angļu val.)

Periodicals and other sources:

1. www.wikipedia.org.

Notes:

Students are provided with lecture materials in electronic form, which they can use during lectures. The workshops prioritize homework assignments, the solutions of which students must complete at home.