Partial differential equations

Course title	**Partial differential equations**
Course code	Mate6012
Branch of science	Mathematics
Science sub-sector	Differential equations, mathematical physics
Credit points	4
ECTS credit points	6
Total contact hours	64
Number of lectures	48
Seminars and practical work hours	16

Course developer(s)
Dr.math., Associate professor Ināra Jermačenko
(Inara Yermachenko http://de.du.lv/mrc/ij.htm)

Prior knowledge (title of the course, part of the program where the course to learn)
Mate5015

Course summary
The course is designed for Master's study program "Mathematics" (sub-branch "Differential equations") students.
The course aims to acquaint students with the theory of partial differential equations and the main analytical solving methods of partial differential equations.

Results
- get an idea of partial differential equations and their types;
- be able to solve analytically quasi-linear first-order partial differential equations;
- be able to reduce second-order linear partial differential equations to canonical form;
- be able to find d'Alembert solution of the Cauchy problem for the one-dimensional wave equation;
- be able to apply the method of separation of variables to solve partial differential equations.

Course content

Lectures – 48 contact hours, seminars – 16 contact hours

Modelling phenomena of the real world by partial differential equations. First-order partial differential equations. Classification of second-order partial differential equations. Cauchy-Kowalewskaya theorem. Hyperbolic equations. Parabolic equations. Elliptic equations. Wave equation. D'Alembert solution. Heat conduction equation. Maximum principle. Laplace equation. Poisson's equation. Harmonic functions. Green's function. Separation of variables.

Course plan

Lectures – 48 contact hours, seminars – 16 contact hours

Lecture topics:

1. Modelling phenomena of the real world by partial differential equations. The classification of partial differential equations.
2. First-order partial differential equations. General solution. Cauchy problem.
3. Transport equation. The method of characteristics.
4. The method of separation of variables for the first order partial differential equations.
5. Non-linear first order partial differential equations and methods of their solving.
6. The classification of second-order partial differential equations. Characteristic equation. Reduction to canonical form.
7. Solutions of the local Cauchy problem. The Cauchy-Kowalewskaya theorem.
8. One-dimensional wave equation. The Cauchy problem for the homogeneous wave equation. Physical interpretation of the d'Alembert solution.
9. Mixed problems for semi-infinite strings. Initial conditions and boundary conditions. The continuation method.
10. Mixed problems for finite strings.
11. The method of separation of variables (Fourier method) for the wave equation. The principle of superposition.
12. Nonhomogeneous wave equations.
13. The Cauchy problem for the homogeneous wave equation in central symmetry case.
14. Heat conduction equation. Cauchy problem. Boundary value problems.
15. The Maximum principle of the heat conduction equation.
16. The method of separation of variables for the heat conduction problem.
17. Nonhomogeneous heat conduction equations.
18. Stationary processes. Laplace equation. Fundamental solution.
19. Properties of harmonic functions. Green's formulas.
20. The Dirichlet problem for the Laplace equation on a rectangle. The method of separation of variables.
21. Laplace equation in polar coordinates. Dirichlet problem on a disk.
22. The Dirichlet problem for the Poisson's equation.
23. The Neumann problem for the Laplace equation on the plane. Boundary value problem with the third type of boundary conditions.
24. Boundary value problems in three-dimensional space.

Seminar topics:

1. Solving of first-order partial differential equations by the method of characteristics.
2. Solving of first-order partial differential equations by the method of separation of variables.
3. Solving methods of non-linear first-order partial differential equations.
4. Reduction of second-order partial differential equations to canonical form.
5. One-dimensional wave equation. D'Alembert solution.
6. The method of separation of variables for the wave equations.
7. Heat conduction equation. The method of separation of variables for heat conduction problems.
8. The Dirichlet problem for the Laplace equation on a rectangle and a disk.

Students' independent work:

During the semester must fulfill six independent works.
Each of the master students' independent work options is individual. A report on the individual performance of the work is to be submitted by the beginning of the session.

Requirements for credits
The course provides the knowledge, skills, skills acquisition.
During the study course examination form – exam.
At the exam students are offered two theoretical questions, completed independent assignments is taken into account in the assessment exam.
Requirements of the course learning – regular attendance and active work in 40%, independent work execution of 50%, exam 10%.
Used in study methods and forms – lectures, seminars, independent work.
The course is acquired in Latvian.

Basic Training

1. A. Buiķis. Matemātiskās fizikas vienādojumi, Rīga, Latvijas Universitāte, 2002.
2. H. Kalis. Matemātiskās fizikas vienādojumi, klasifikācija un izvedumi. Stīgas svarstības vienādojums, Rīga, Latvijas Universitāte, 1992.
3. R.P. Agarwal, D. O'Regan. Ordinary and Partial Differential Equations: With Special Functions, Fourier Series, and Boundary Value Problems, Springer, 2008.
4. L. Debnath, T. Myint-U. Linear Partial Differential Equations for Scientists and Engineers, Birkhauser Boston, 2007.
5. В.А. Байков, Жибер А.В. Уравнения математической физики, Москва – Ижевск, 2003.
6. Л.К. Мартинсон, Ю.И. Малов. Дифференциальные уравнения математической физики, МГТУ, 2002.

Further reading

1. H. Kalis. Siltuma vadīšanas un Puasona vienādojumi, Rīga, Latvijas Universitāte, 1992.
2. E. Riekstiņš. Matemātiskās fizikas vienādojumi, Rīga, Latvijas valsts izdevniecība, 1964.
3. P. Drabek, G. Holubova. Elements of Partial Differential Equations, de Gruyter, 2007.
4. L.C. Evans. Partial Differential Equations, AMS, 1998.
5. R. Haberman. Elementary Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, Prentice Hall, 1987.
6. P.V. O'Neil, Beginning Partial Differential Equations, John Wiley & Sons, 1999.
7.Y. Pinchover, J. Rubinstein. An introduction to partial differential equations, Cambridge University Press, 2005.
8. M.A. Pinsky. Partial Differential Equations and Boundary-value Problems with Applications, AMS, 2011.
9.А.И. Комеч. Практическое решение уравнений математической физики, Москва, 1993.
10.С. Фарлоу. Уравнения с частными производными для научных работников и инженеров, Мир, 1985.

Periodicals and other sources of information

Remarks

We identify programs and portions (A, B, C, D) adheres to this course

Master's study program "Mathematics" (sub-branch "Differential equations") Part A.